Fun With Natural Numbers

In this problem we will have some fun with the Natural numbers (1, 2, 3, 4, …). You are to implement four methods in the FunWithNaturalNumbers class. The four methods are summation(int n), prod(int n), groupEm(int n), and sumThoseGroups().

The summation(int n) method returns the largest value m such that 1+2+3+…+m <= n. For example:

summation(1+2+3) = summation(6) = 3: since 1+2+3 = 6

summation(50) = 9: since 1+2+3+4+5+6+7+8+9 = 45

and 1+2+3+4+5+6+7+8+9+10 = 55

summation(91) = 13: since 1+2+3+4+5+6+7+8+9+10+11+12+13 = 91

The following code shows the results of the summation(n) method.

|  |  |
| --- | --- |
| The following code | Returns |
| FunWithNaturalNumbers.summation(1+2+3); | 3 |
| FunWithNaturalNumbers.summation(50); | 9 |
| FunWithNaturalNumbers.summation(91); | 13 |

The prod(int n) method returns the largest value m such that 1\*2\*3\*4\*m <= n(n+1). For example:

prod(24) = 5: 1\*2\*3\*4\*5 = 120 <= 600 = 24\*25

and 1\*2\*3\*4\*5\*6 = 720 > 600 = 24\*25

prod(99) = 7: 1\*2\*3\*…\*6\*7 = 5040 <= 9900 = 99\*100

and 1\*2\*3\*…\*6\*7\*8 = 40320 > 9900 = 99\*100

prod(2318) = 10: 1\*2\*3\*…\*9\*10 = 3628800 <= 5375442 = 2318\*2319

and 1\*2\*3\*…\*10\*11 = 3991680 > 5375442 = 2318\*2319

The following code shows the results of the prod(n) method.

|  |  |
| --- | --- |
| The following code | Returns |
| FunWithNaturalNumbers.prod(24); | 5 |
| FunWithNaturalNumbers.prod(99); | 7 |
| FunWithNaturalNumbers.prod(2318); | 10 |

The groupEm(int n) method is used to help demonstrates just how amazing Natural numbers are. Write out the Natural Numbers:

1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, ….

Now arrange them into groups of 1 number, 2 numbers, 3 numbers, and so on:

1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, ….

And cross out the 2nd, 4th, 6th and all other even numbered groups.

1, ~~2 , 3~~, 4, 5, 6, ~~7, 8, 9, 10~~, 11, 12, 13, 14, 15, ~~16, 17, 18, 19, 20, 21~~, 22, 23, 24, 25, 26, 27, 28, ~~29, 30,~~ ….

So you are left with:

1, 4, 5, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 37, 38, 39, 40, 41, 42, 43, 44, 45, …..

The groupEm(int n) method should return a List containing the corresponding group of numbers. For example:

groupEm(1) returns the List containing the Integer 1.

groupEm(2) returns the List containing the Integers 4, 5, and 6.

groupEm(3) returns the List containing the Integers 11, 12, 13, 14, and 15

The following code shows the results of the groupEm(n) method.

|  |  |
| --- | --- |
| The following code | Returns |
| List<Integer> ans = FunWithNaturalNumbers.groupEm(2); |  |
| ans.size(); | 3 |
| ans.contains(new Integer(4)); | true |
| ans.contains(new Integer(5)); | true |
| ans.contains(new Integer(6)); | true |

The following code shows the results of the groupEm(n) method.

|  |  |
| --- | --- |
| The following code | Returns |
| List<Integer> ans = FunWithNaturalNumbers.groupEm(3); |  |
| ans.size(); | 5 |
| ans.contains(new Integer(11)); | true |
| ans.contains(new Integer(12)); | true |
| ans.contains(new Integer(13)); | true |
| ans.contains(new Integer(14)); | true |
| ans.contains(new Integer(15)); | true |

The sumThoseGroups(int n) method demonstrates just how amazing Natural numbers are. The sumThoseGroups(int n) method returns the sum all the ints from group 1 up to and including group n as defined by the groupEm method. For example:

sumThoseGroups(1) returns the value 1.

sumThoseGroups(2) returns the value 16 = groupEm(1) + groupEm(2) = 1 + (4 + 5 +6).

sumThoseGroups(3) returns the value 81 = groupEm(1) + groupEm(2) + groupEm(3)  
 = 1 + (4 + 5 +6) + (11 + 12 + 13 + 14 + 15)

Oh yea, how does this demonstrate that Natural numbers are amazing. I leave that up to you to figure out ☺.

The following code shows the results of the sumThoseGroups(n) method.

|  |  |
| --- | --- |
| The following code | Returns |
| FunWithNaturalNumbers.sumThoseGroups(1); | 1 |
| FunWithNaturalNumbers.sumThoseGroups(2); | 16 = 1+(4+5+6) |
| FunWithNaturalNumbers.sumThoseGroups(3); | 81 =  1 + (4+5+6) + (11+12+13+14+15) |